

the tape material was 1.637×10^3 MPa and Poisson's ratio was 0.4. Figures 6-1, -2 and -3 depict the computed distribution of strains in the 10-layer roll of tape, which correspond to the stresses shown in Figures 5A-1, -2 and -3.

It can be seen that for 10 wraps both circumferential and radial strains are relatively small and circumferential strain monotonically decreases with an increase in radius (Figure 6-1). For 50 wraps, the trend is similar, however, the level of strain is higher than that for 10 wraps. Also, the role of radial strain has increased compared to circumferential strain, especially in the first few layers (Figure 6-2).

Additional computation revealed a parabolic upturn in the circumferential strain curve for 460 wraps (Figure 6-3). The circumferential strain curve shows a monotonic increase in a major portion of the curve after approximately 100 layers. In the first few layers, the amplitude of radial strain is above the level of circumferential strain.

Assuming that the final distribution of EFL along the roll radius is a difference between the initial constant EFL_0 value before reeling, and the circumferential strain induced by reeling, one can obtain the EFL distribution shown in Figures 7-1 and -2. In these calculations, the initial constant value of EFL_0 was taken to be 0.1.

As can be seen from Figures 7-1 and -2 at a constant take up stress (i.e. draw tension), 10 layers produce a relatively small variation in the EFL distribution (Ex. 1). This variation significantly increases for 460 layers (Ex. 2). In both cases, the most dramatic change in EFL (curve Ex. 1) takes place in the zone close to the spool core. Computation for 2500 layers revealed enhanced sharpness in the EFL curvature at the spool core surface (Figure 7-2).

To further investigate the relationship of stress and strain on EFL of reeled or wrapped buffer tubes, analysis under a second analytical model was also conducted. In this

modeling variable wrapping stress and relative stiffness of the core of the reel and the wrapped material were analyzed to determine a method to obtain a relatively constant EFL throughout the length of a reeled buffer tube. In this analysis, an existing model was modified for constant and monotonically decaying take-up tension.

- 5 In Wolfermann, W. and Schröder D. (1987), "Web Forces and Internal Tensions for the Winding of an Elastic Web," *International Conf. "Winding Technology 1987"*, Stockholm, Sweden, 1987, S. 25 - 37, which is incorporated herein by reference, the stress distribution in rolled materials based on the model of a circular ring that is shrunk by a winding tension, σ_w , was analyzed. The influence of a controlled variation of winding
- 10 tension on the stress distribution was also investigated. For anisotropic materials, equations for circumferential and radial stresses were presented in the following form:

$$\sigma_{\theta} = \sigma_w - \left[\delta + \gamma \beta \left(\frac{r}{r_o} \right)^{2\kappa} \right] \Delta\sigma ; \quad (4.1)$$

$$\sigma_r = \left[\beta \left(\frac{r}{r_o} \right)^{2\kappa} - 1 \right] \Delta\sigma . \quad (4.2)$$

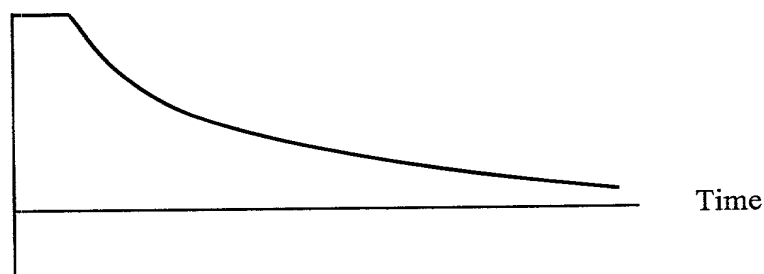
$$\text{where } \Delta\sigma = \frac{1}{r^{\delta+1}} \int_{r_o}^R \frac{\sigma_w r^{\delta}}{\beta \left(\frac{r}{r_o} \right)^{2\kappa} - 1} dr ,$$

- 15 parameters γ , δ and κ represent anisotropic properties of the wrapped material, and β is the parameter relating stiffness of the wrapped material to that of the core.

When Young's modulus of the core material is much higher than that of the wrapped material, $\beta \approx -1$, and can be as low as -2 for very stiff core materials. This range of values

for β was considered by Wolferman and Schroder to model the stress distribution in paper rolled on a steel core. For the case of paper rolled on a paper roll, the authors suggested $\beta = 2$. According to Wolferman and Schroder, radially shrinking layers influence the circumferential stresses in the middle part of the roll resulting in compressive circumferential stresses, with the largest compressive stresses achieved when $\beta = 2$. Thus, it was recommended to use cores made of hard materials. Also, the authors recommended use of a two-stage function for wrapping stress to reduce the range of the stress variation. Initially, wrapping should be performed under a constant high level of wrapping stress. After a certain number of wraps, the wrapping stress should be monotonically reduced as shown in the figure below:

Take-up tension



Two-stage Variable Take-up Tension from Wolfermann and Schroder.

For isotropic materials, $\gamma = \delta = \kappa = 1$ and Equations 4.1 and 4.2 can be simplified to

$$\sigma_{\theta} = \sigma_w - \frac{r_o^2 + \beta r^2}{r^2} \int_{r_o}^R \frac{\sigma_w r}{\beta r^2 - r_o^2} dr, \quad (4.3)$$

$$\sigma_r = \frac{r_o^2 - \beta r^2}{r^2} \int_{r_o}^R \frac{\sigma_w r}{\beta r^2 - r_o^2} dr. \quad (4.4)$$